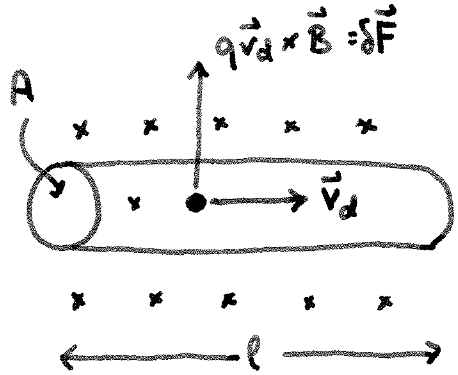
L14

Last time, we learnt that the magnetic force on a charged particle is $q\vec{v}\times\vec{B}$. When we add up the forces on every moving charge, this creates the force on a wire carrying a current. It is this force which drives electric motors. We are now going to study it in more detail. This idea will also enable us to understand why the like poles of a magnet repel one-another. At the end of our class today we'll learn about an effect discovered by the 19^{th} century physicist Edwin Hall, which enables us to actually measure the density & sign of the charge of current carriers

inside a conductor.

27.6 Force on a wire

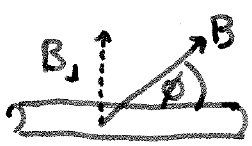


Suppose we add up the total magnetic force on the wire.

$$F = \overbrace{nA\ell}^{\#\text{ charges}} \underbrace{(q v_d B)}_{\text{force per charge}} = \underbrace{\overbrace{nqv}^{J}\ A}_{I}\ \ell B$$

$$F = I\ell B$$

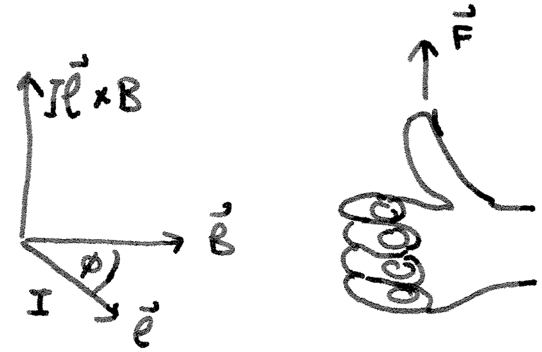
Actually, if the field is at an angle ϕ to the wire

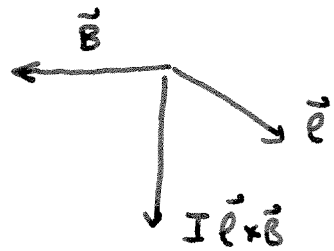


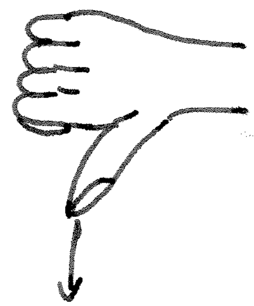
$$F = I\ell B_\perp = I\ell B\sin\theta$$

The direction is perpendicular to the wire & the field

$$\boxed{\vec{F} = I\,\vec{\ell} \times \vec{B}}$$

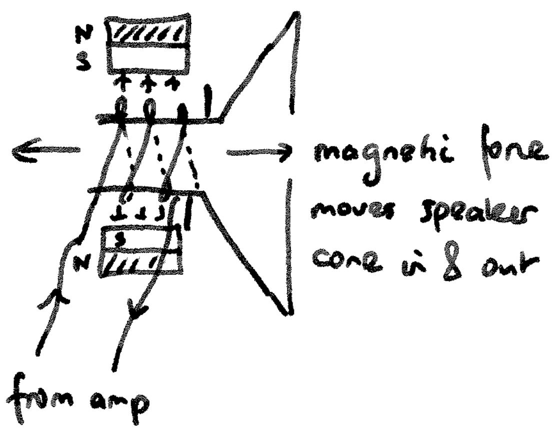






Applications

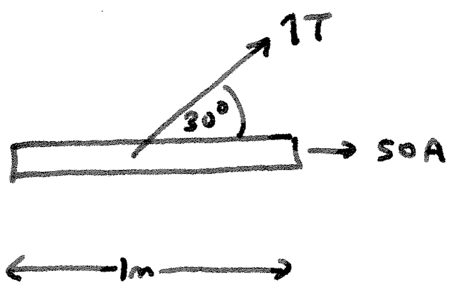
- loudspeaker.



- electric motors.

e.g. 1 Tesla @ 30° to a 1m wire running along x direction, carrying 50A.

What is the force on the wire?



$$F = I \ell B \sin\theta = 50\text{A} \times 1\text{m} \times 1\text{T} \times \sin 30^\circ = \underline{25\text{N}}$$

$$\vec{F} = I \vec{\ell} \times \vec{B}$$

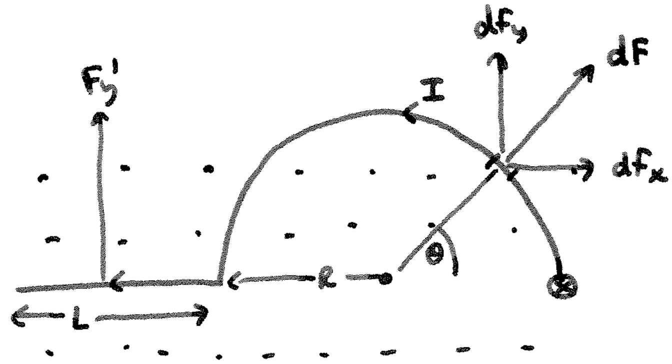
$$\vec{\ell} = 1\hat{\imath}$$

$$\vec{B} = (\cos 30\,\hat{\imath} + \sin 30\,\hat{\jmath}) = \left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right)$$

$$\vec{F} = 50\,\hat{\imath} \times \left(\frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}\right) = 25\,(\hat{\imath} \times \hat{\jmath}) = 25\,\text{N}\,\hat{k}$$

The direction is $\perp$ to the wire.

e.g Force on a curved conductor



$$dF = I\,dl\,B$$

$$dF_y = I\,dl\,B\sin\theta = IRB\sin\theta\,d\theta$$

$$F_y\,(\text{curved}) = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB$$

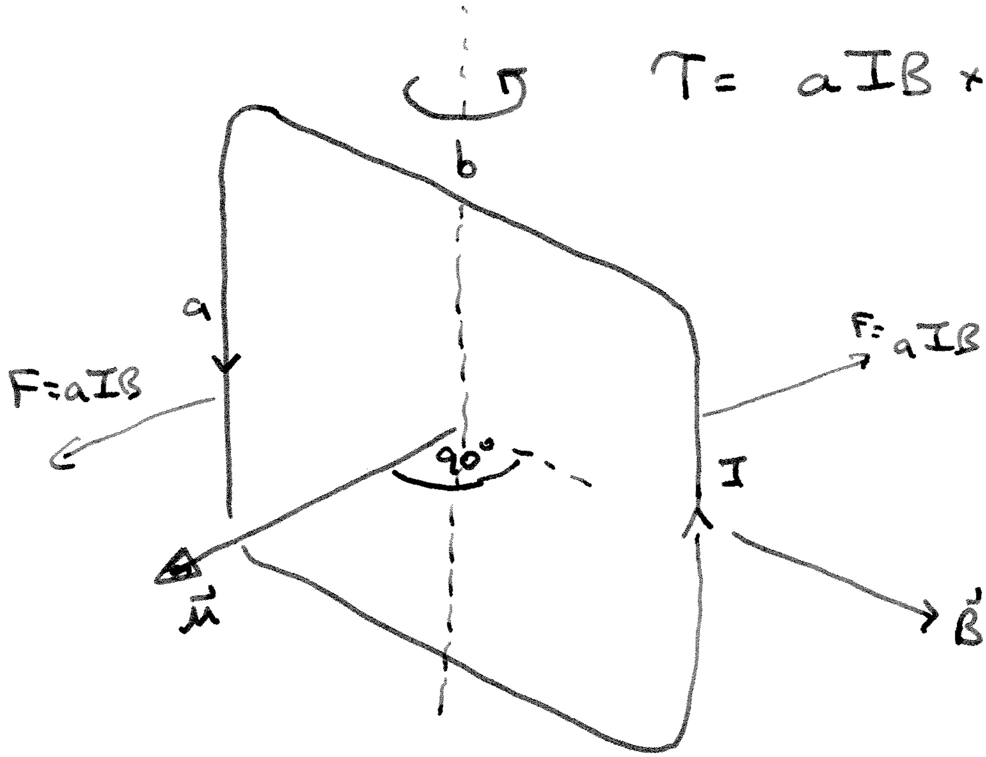
$$F_y\,(\text{straight}) = F_y' = ILB$$

$$F_y\,(\text{total}) = IB(L+2R) \qquad F_x = 0 \text{ (by symmetry)}$$

$$\boxed{\vec{F} = IB(L+2R)\,\hat{j}}$$

MAGNETIC DIPOLE

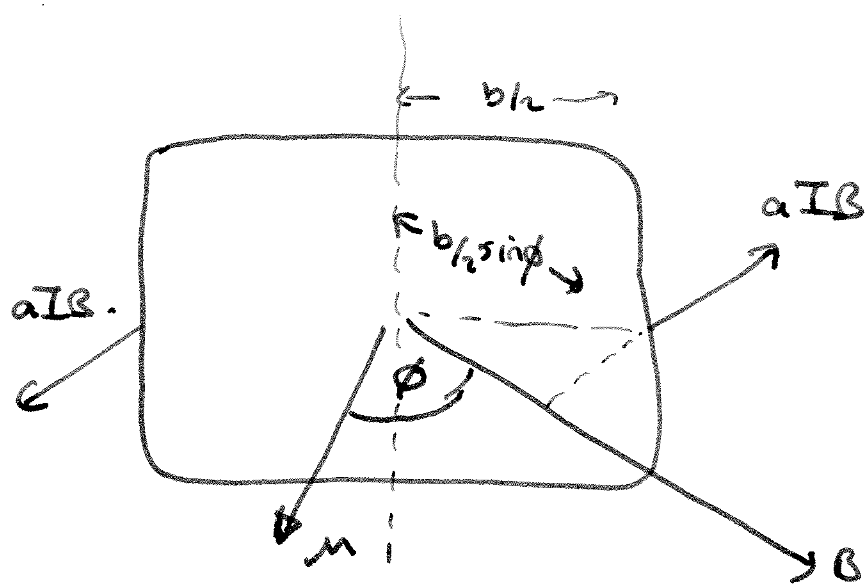
$$\tau = aIB \times b = \underline{\underline{\mu B}}$$



$$\boxed{\mu = IA}$$

$$\vec{\mu} = \mu \hat{n}$$

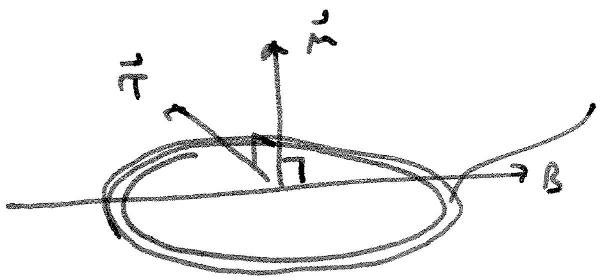
$$\tau = \mu B \sin\phi$$



e.g. Coil with 50 turns, $I = 2A$, radius 5 cm, $B = 2T$

How big is the magnetic moment μ?

How big is the torque on the coil when the magnetic field lies in the plane of the coil?



$$\mu_{TOT} = NIA$$
$$= 50 \times 2A \times \pi (0.05)^2$$
$$= 7.85 \times 10^{-1}\ Am^2$$
$$= 0.785\ Am^2$$

$$\tau = \mu B = 0.785 \times 2 = \underline{1.57\ Nm}.$$

What is the change in P.E rotating from $\phi = 90°$ to $\phi = 0°$

$$U_1 = -\mu_{total} B \cos\phi_1 = 0$$

$$U_2 = -\mu_{total} B \cos\phi_2 = -\mu_{total} B = -1.57\ J$$

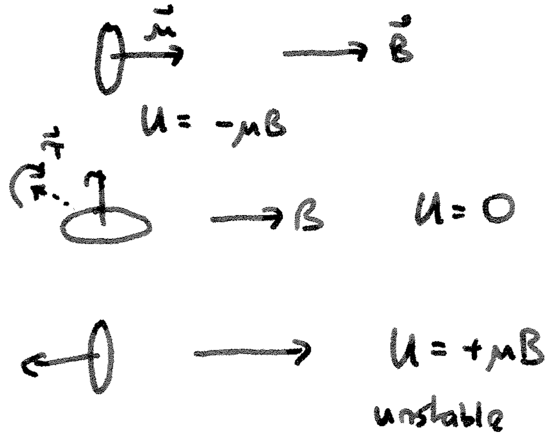
$$\Delta U = -1.57\ J.$$

Energy of magnetic dipole in a field

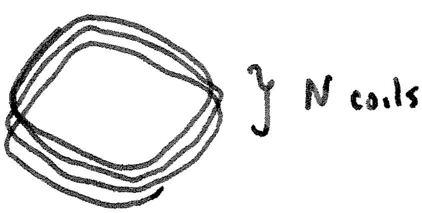
Work done in changing ϕ from $\phi = \phi_1$ to $\phi = \phi_2$

$$W = U_2 - U_1 = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \mu B \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = \Big[-\mu B \cos\theta \Big]_{\theta_1}^{\theta_2}$$

$$U = -\mu B \cos\theta = \underline{\underline{-\vec{\mu} \cdot \vec{B}}}$$



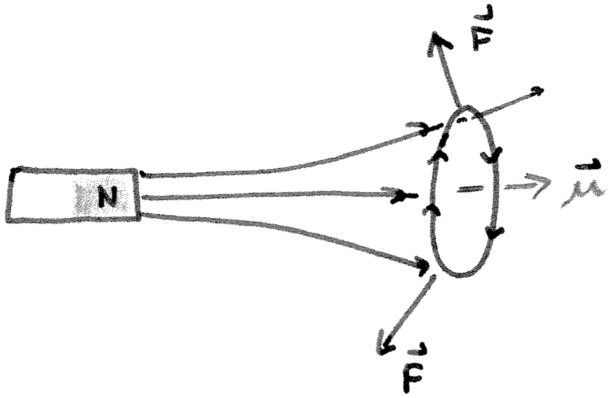
Many turns on coil



$$\tau = N \times IAB \sin\phi$$

Galvanometer deflection $\propto NIAB \sin\phi$.

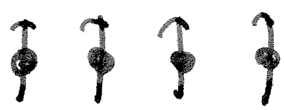
Dipole in a non uniform field.



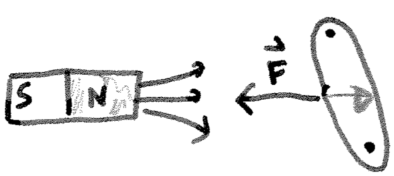
attracted towards higher field

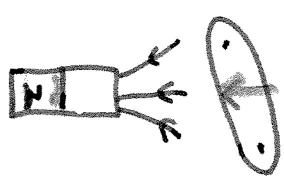
repelled.

Permanent magnet — contains <u>aligned</u> electron spins.

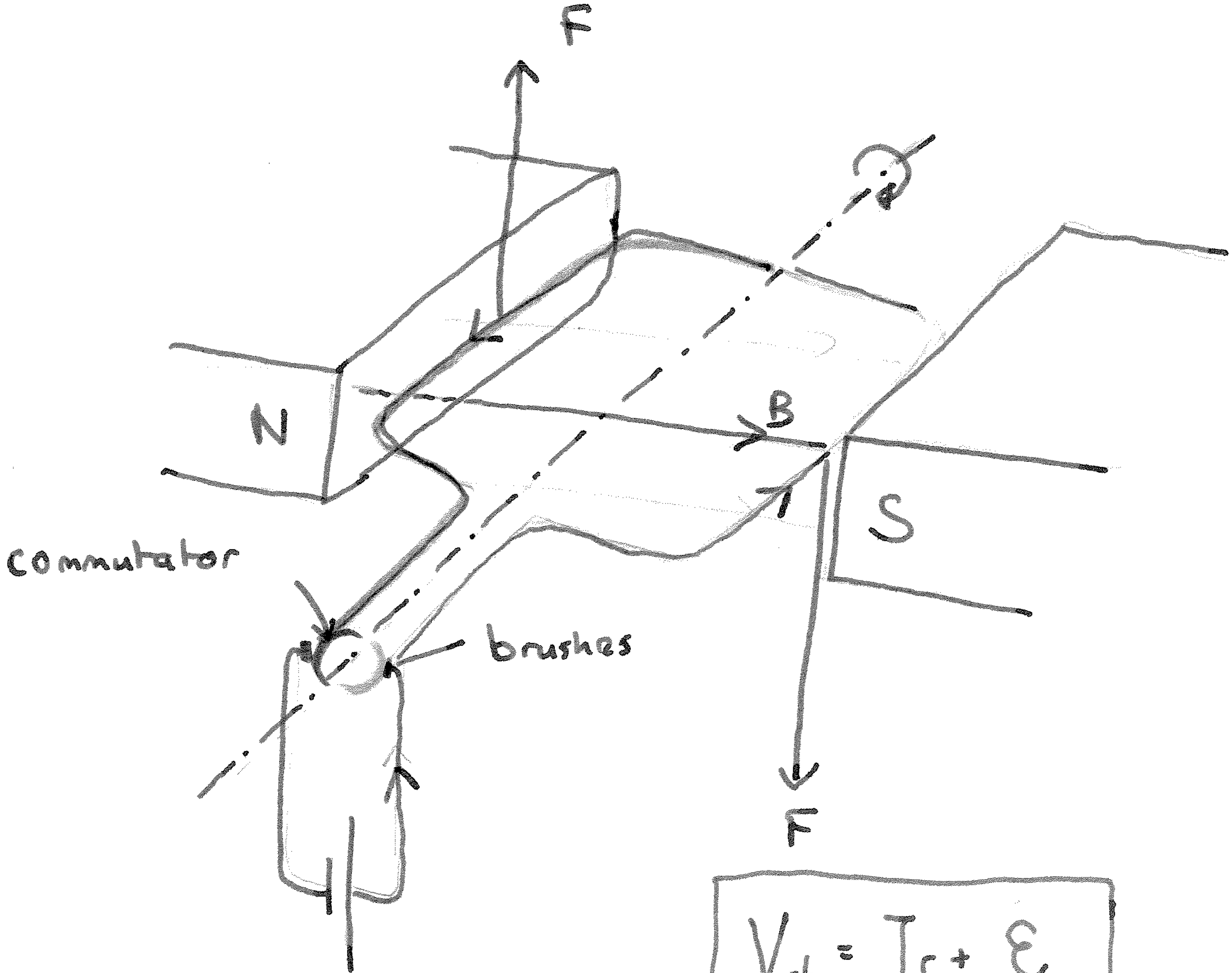


Ferromagnet.

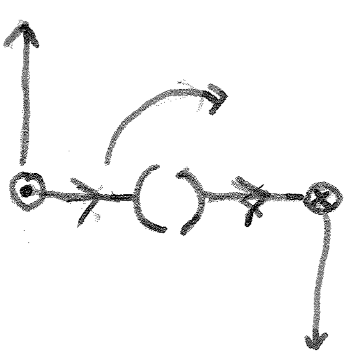




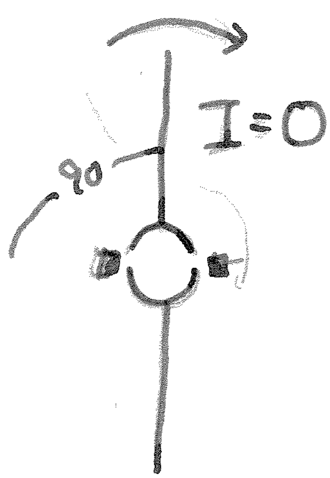
D.C. Motor



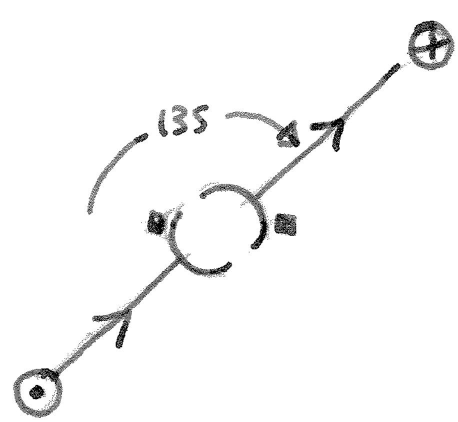
$$\boxed{V_{ab} = Ir + \underset{\text{BACK EMF}}{\mathcal{E}}}$$



0°

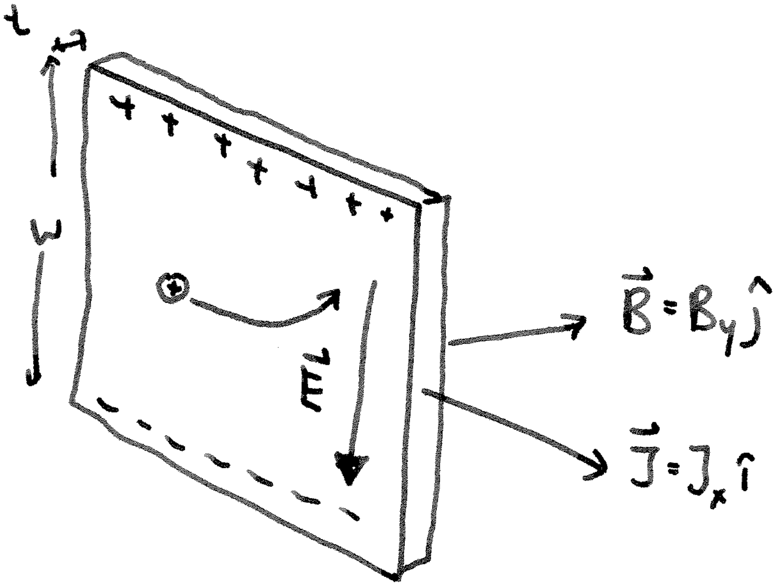


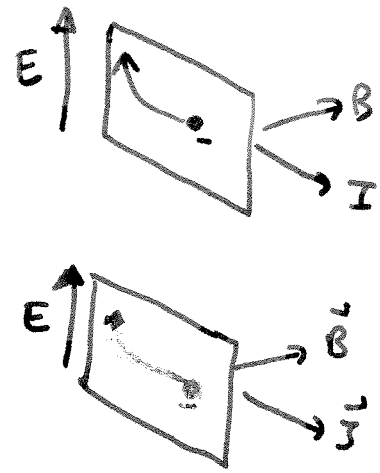
90°



135°

HALL EFFECT





$$q E_z + q v_d B_y = 0$$

$$\Rightarrow \quad E_z = -v_d B_y$$

$$= -B_y \frac{J_x}{nq}$$

$$\left(nq = -\frac{J_x}{E_z} B_y \right)$$

$$\boxed{\frac{1}{nq} = R_H}$$ Hall constant

e.g. $t = 2mm$ $w = 1.5cm$ $\Delta V = 0.81 mV$

$I = 75A$

$$J_x = \frac{I}{A} = \frac{75A}{2\times10^{-3}m \times 1.5\times10^{-2}m} = 2.5\times10^{6} A/m^2$$

$$E_z = \frac{\Delta V}{t} = \frac{0.81\times10^{-6}}{1.5\times10^{-2}} = 5.4\times10^{-5} V/m$$

$$n = -\frac{J_x B_y}{qE_z} = -\frac{(2.5\times10^{6}A/m^2)(1T)}{(-1.6\times10^{-19}C)(5.4\times10^{-5}V/m)}$$

$$= \underline{11.6\times10^{28} m^{-3}}$$