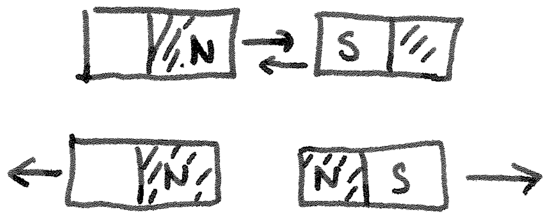
13 MAGNETIC FIELD + FORCES

Magnetism + Magnetic forces have intrigued humankind for millenia, yet it is only in the last 200 years that we have come to understand magnetism, and only in the last 100 years that our technology has learnt to harness the forces of magnetism. Today we use magnetic forces literally everywhere – they are the driving force of electric motors, loudspeakers – they are vital for communication, printers, disk drives.

Once again we're going to use the concept of a field to describe the forces created by magnetism. Today we're going to learn the force on a charged particle created by a field. Next time – next week – we will learn how a moving particle creates a magnetic field.

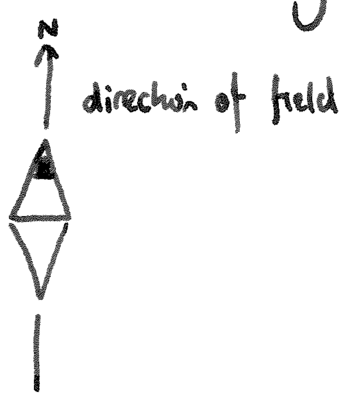
27.1 Magnetism

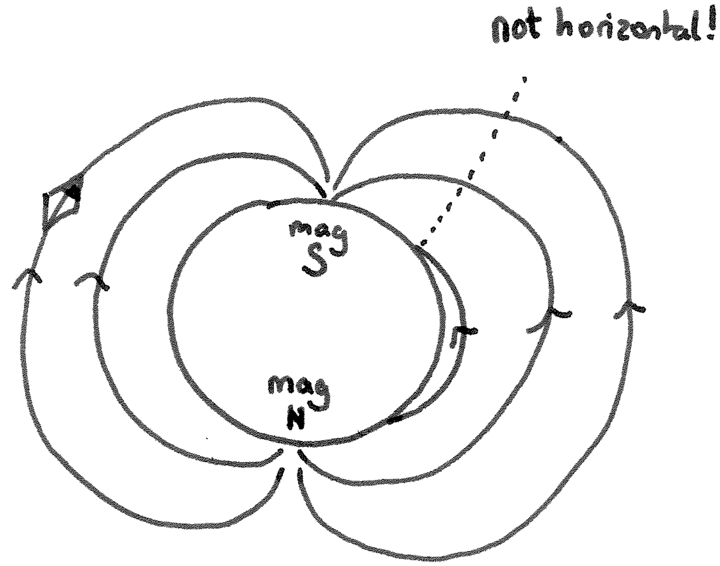
{ Opposite poles attract
{ Like poles repel



- Both poles attract iron.

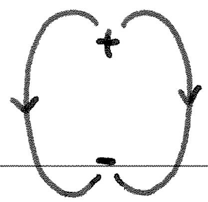
- Compass aligns along a given direction — the direction of the magnetic field



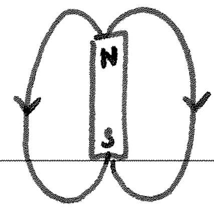


- Notice that the N magnetic pole is at the S geographic pole !!

- Analogous to electric charges

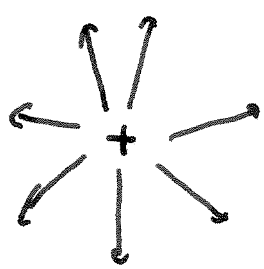


electric field

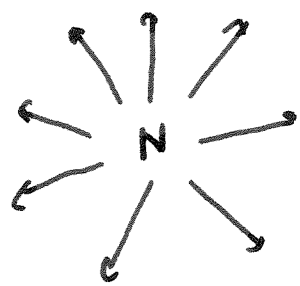


magnetic field

However – isolated magnetic charges or "monopoles" have never been found, despite extensive searches.



charge ✓



monopole X

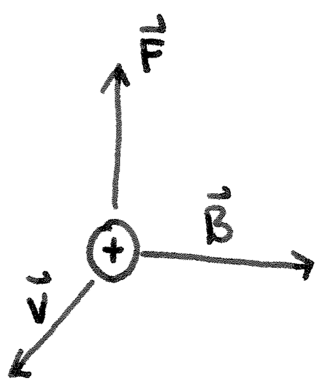
27.2

Recall:

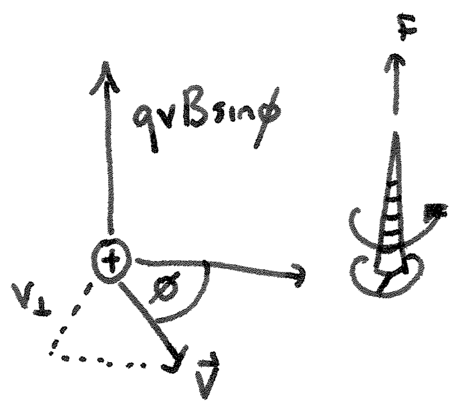
> charges → electric fields
>
> electric fields → electric force $\vec{F} = q_0 \vec{E}$

Magnetic field:

> <u>moving</u> charges → magnetic fields
>
> magnetic field → force on moving charge



$$F = |q| v_{\perp} B = |q| v B \sin\phi$$



$$\vec{F} = q \vec{v} \times \vec{B}$$

Force (magnetic) on moving charged particle

$$1 \text{ Tesla} = 1T = 1N/A \cdot m$$

(c.f 1 Gauss = 10^{-4} T.)

Largest steady fields ~ 45 T
(NHMFL, Florida)

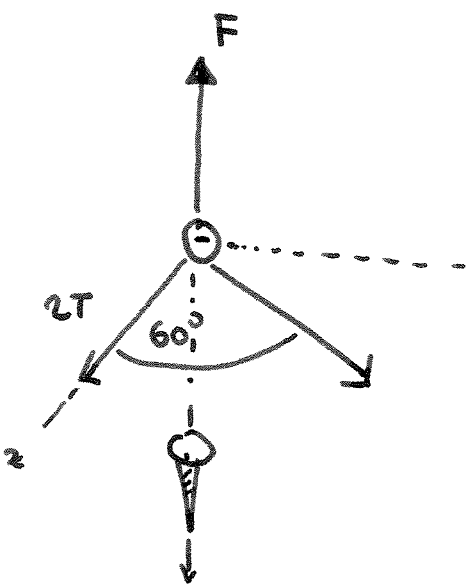
Pulsed fields up to 120 T

Neutron Star 10^8 T !

Combined electric & magnetic forces

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

e.g. Beam of electrons 1/100 speed of light at 60° to z axis in zx plane. Magnetic field of 2T along z axis. What is the direction & magnitude of the force?



$$F = |q| v B \sin\phi$$

$$= (1.6\times10^{-19}) \times (3\times10^{6}\,\text{m/s}) \times 2T \times \sin 60^\circ$$

$$= 4.8\times10^{-13}\ \text{N}$$

vectors

$$\vec{F} = q\,\vec{v}\times\vec{B}$$

$$= (-1.6\times10^{-19}\text{C})\left((\times 3\times10^{6}\,\text{m/s})(\cos 60^\circ\,\hat{z} + \sin 60^\circ\,\hat{x})\right) \times (2T\,\hat{z})$$

$\hat{z}\times\hat{z} = 0$
$\hat{z}\times\hat{x} = \hat{y}$
$\hat{x}\times\hat{z} = -\hat{y}$

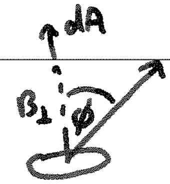
$$= -1.6\times10^{-19}\,\sin 60^\circ \times 3\times10^{6} \times 2\,(-\hat{y})$$

$$= \underline{4.8\times10^{-13}\,\text{N}}\ \hat{y}.$$

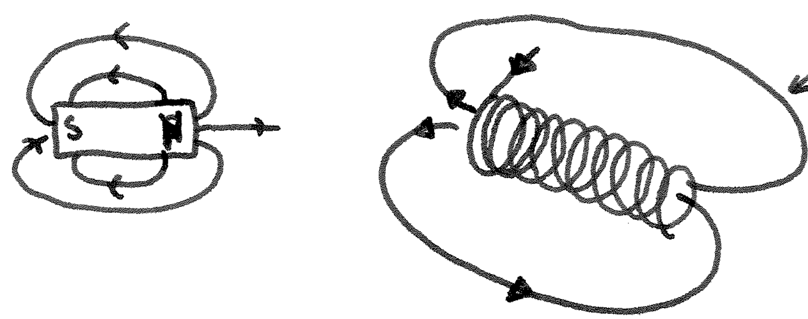
27.3 Field lines + Flux.

Magnetic flux $d\Phi_B = B_\perp \, dA = B\cos\phi \, dA$

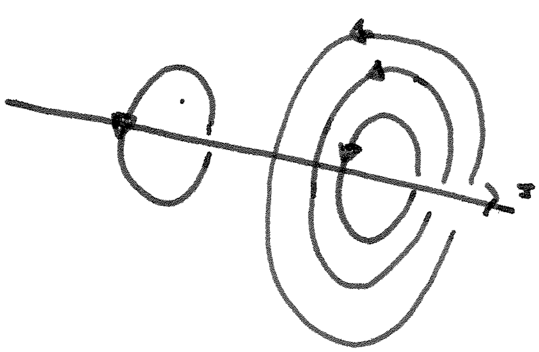
$= \vec{B} \cdot d\vec{A}$



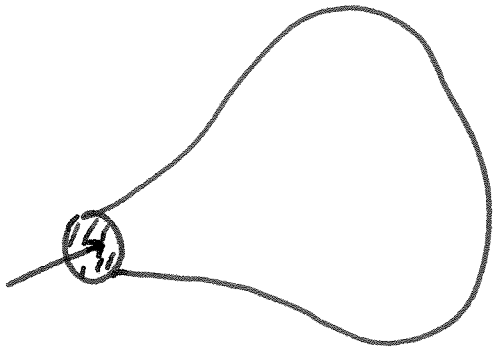
Gauss' law $$\int \vec{B} \cdot d\vec{A} = 0$$ no monopoles.



lines of force never end anywhere.



e.g. a)



Flux in = 0.1 Wb

Q Flux out through remainder of surface ?

A. 0.1 Wb

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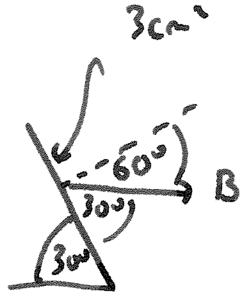
b)

$\phi = 0.9\,mWb$

$A = 3\,cm^2$



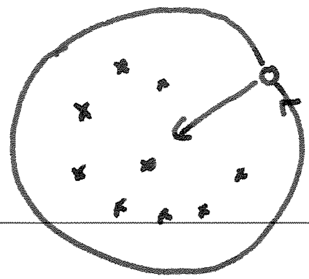
What is the field ?

$\phi = BA\cos 60°$

$$B = \frac{0.9 \times 10^{-3}\,Wb}{(3 \times 10^{-4}\,m^2)\cos 60°} = \underline{6\,T}$$

13.10

27.4 Motion of charge particles in a field



$F = qBv$ ⊥ to motion

$= m\left(\frac{v^2}{R}\right)$

$\Rightarrow$ $R = \frac{mv}{|q|B}$

$\Rightarrow$ $\omega = \frac{v}{R} = \frac{|q|B}{m}$ = cyclotron frequency.

e.g. electrons accelerated in 300V, move in 5cm radius circle

a) How fast are they moving?

b) What is field?

c) What is the cyclotron frequency?

a) $qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 300}{9.1 \times 10^{-31}}}$

$= 1.03 \times 10^7$ m/s

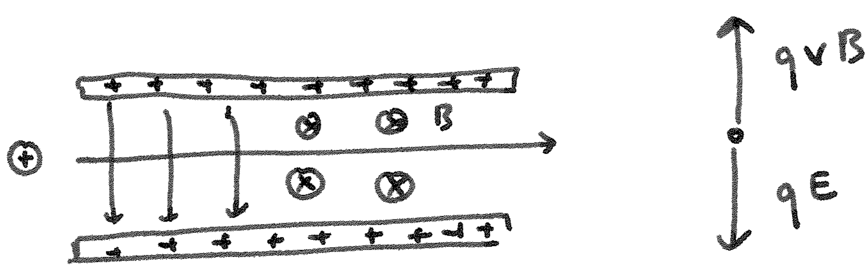
$\simeq \frac{1}{30}$ th c.

b) $$B = \frac{mv}{qR} = \frac{9.1\times10^{-31}\times 1.03\times10^{7}}{1.6\times10^{-19}\times 5\times10^{-2}} = 1.17\times10^{-3}\ \mathrm{T}$$

$(\equiv 11.7\ \mathrm{G})$

c) $$\omega = \frac{v}{R} = \frac{qB}{m} = \frac{1.03\times10^{7}}{5\times10^{-2}} = \underline{2.06\times10^{8}\ \mathrm{Hz}}$$

27.5 Velocity selector



$$qvB = qE \Rightarrow \boxed{v = \frac{E}{B}}$$

does not depend on charge or mass of particle.

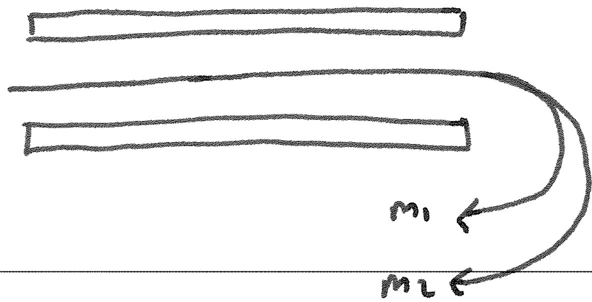
e/m ratio

$$\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}} = \frac{E}{B}$$

$$\Rightarrow \frac{2qV}{m} = \frac{E^2}{B^2} \qquad \boxed{\frac{q}{m} = \frac{E^2}{2VB^2}} \simeq 1.76\times10^{11}\ \mathrm{C/kg}$$

~ 2000 greater than for protons ⇒ NEW PARTICLE (J.J. Thomson).

Mass Spectrometer



$$R = \frac{mv}{qB} \propto m.$$

Contents of beam are separated according to their mass. Large mass ⇒ large radius.