L12

Today we'll talk of two things. We'll begin by discussing how we measure current, voltage & resistance. If you go to Radio Shack or Home Depot, you can pick up a small device called a "multimeter" which measures all these things. What's inside it ?

We'll then turn to take our first look at a "dynamical circuit": a circuit in which the current changes with time. Today we will look at an "RC circuit", a circuit in which a capacitor

& a resistor are connected in series. We'll see that the way a capacitor discharges is very reminiscent of radio-active decay. Each RC circuit has a "half life" — or alternatively a "time constant" — which determines the time required for the charge to decay (by a given factor).

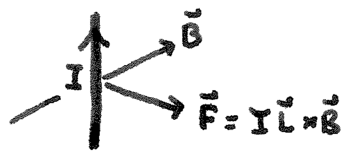
ELECTRICAL INSTRUMENTS

i) • Ammeter
i) • Voltmeter
ii) • Ohmmeter

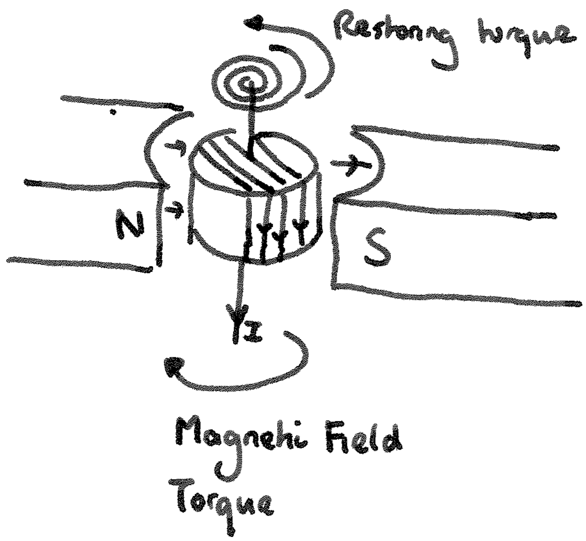
"Multimeter"

iv) • Potentiometer

i) Basic detection system
"Galvanometer"



Magnetic Force on Current.



$\boxed{\text{Deflection} \propto \text{current}}$

Voltage drop

$V = I_{fs} R_c$ (coil)

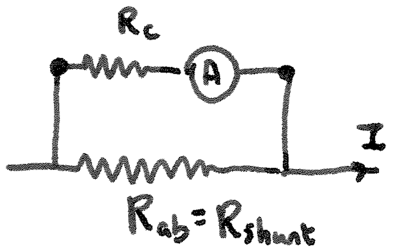
e.g. 1mA 20Ω ⇒ 20mV

Always some voltage drop across a Galvanometer.

Galvanometer can be used for both current + voltage measurement

- Changing current range of Galvanometer

e.g suppose $R_c = 20\Omega$, $I_{fs} = 1mA$ & want to measure a maximum of 10mA.



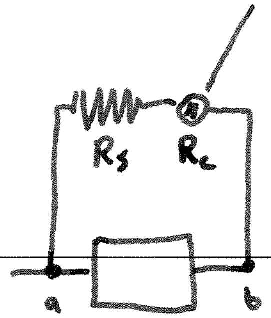
$$V = I_{fs} R_c$$
$$= (I - I_{fs}) R_{ab}$$

$$\Rightarrow \quad I_{fs}(R_c + R_{ab}) = I R_{sh}$$

$$\frac{R_{ab}}{R_{sh}} + 1 = \frac{I}{I_{fs}}$$

$$\frac{I}{I_{fs}} = 10 \qquad \frac{R_{ab}}{R_{sh}} = 9 \qquad R_{ab} = 20\Omega \qquad \underline{\underline{R_{sh} = \frac{20}{9}\Omega}} = 2.\dot{2}\Omega$$

ii) Galvanometer in voltage measuring mode



full swing voltage ↓ full swing current.

$$V_{ab} = (R_s + R_c) I_{sf}$$

e.g: How do we make a voltmeter with a max range 50.0V out of a Galvanometer with $R_c = 20\Omega$ & $I_{fs} = 1mA$?

Require a shunt in series with

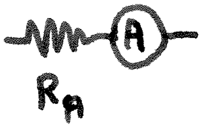
$$50 = V_{ab} = (R_s + R_c) I_{fs}$$

$$\Rightarrow R_s = \frac{V_{ab}}{I_{fs}} - R_c = \frac{50}{10^{-3}} - 20 = 50{,}000 - 20$$

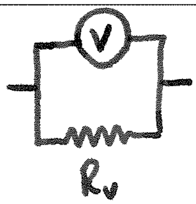
$$R_{sh} = 49{,}980\,\Omega$$

= required shunt resistance.

Ammeters + Voltmeters are never perfect

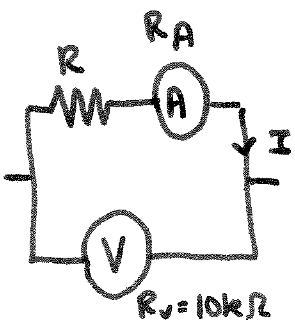


Ammeter + coil resistance



Voltmeter + finite resistance

e.g.



$R_A = 2\Omega$

$R_v = 10k\Omega$

$V = 12V$

$I = 0.1A$

- What is the resistance R ?

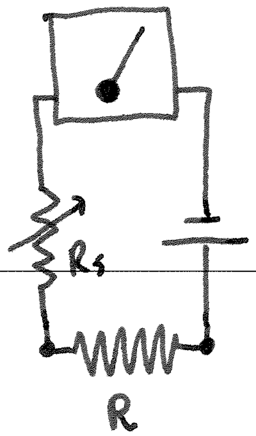
$$V = IR + IR_A \quad \Rightarrow \quad R = \frac{V}{I} - R_A = \frac{12}{0.1} - 2$$

$$= 120 - 2$$

$$= 118\Omega$$

- $P = V_{ab}I = I^2R = 0.1(11.8) = \underline{1.18\,W}$

- Ohmmeter –



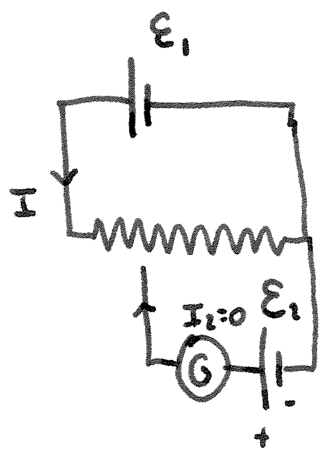
When shorted – full swing $\quad I_{fs} = \dfrac{\mathcal{E}}{R_0}$

With finite R

$$I = \frac{\mathcal{E}}{R_0 + R} = \frac{I_{fs}}{1 + R/R_0}$$

- Potentiometer

Measures EMF of source without drawing any current.

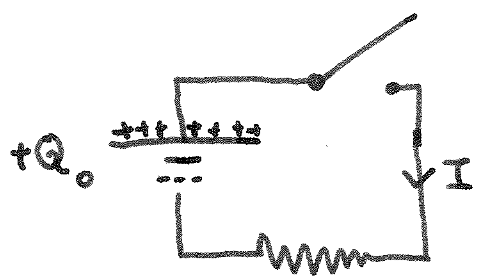


$$I R_{\text{[illegible]}} = \mathcal{E}_1$$

$$I R_{ab} = \mathcal{E}_2 \quad \text{when } I_2 = 0.$$

26.4 R-C Circuits

When we discharge, or charge up a capacitor, we have to deal with time-dependent currents. Charge up a capacitor to $Q_0 = \mathcal{E}_0 C$. Now discharge it



DISCHARGE

Kirchoff $\quad \frac{Q_0}{C} - IR = 0$

$I = \frac{dQ}{dt}$

$$\boxed{Q = -CR\frac{dQ}{dt}}$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{CR} \Rightarrow \ln\frac{Q}{Q_0} = -\frac{t}{CR}$$

$$\Rightarrow \frac{Q}{Q_0} = e^{-t/CR}$$

$$\boxed{Q = Q_0 e^{-t/\tau}} \qquad \underline{\tau = CR}$$

$$I = -\frac{dQ}{dt} = \frac{Q_0}{CR} e^{-t/CR} = I_0 e^{-t/CR}$$

I_0 ↑ initial current.

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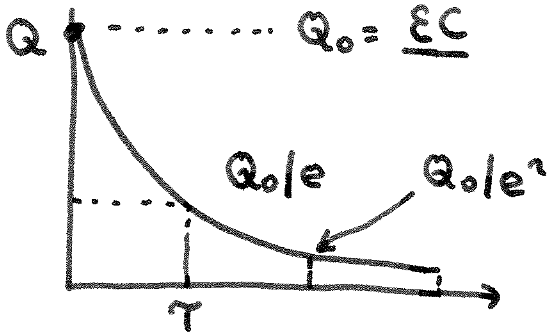
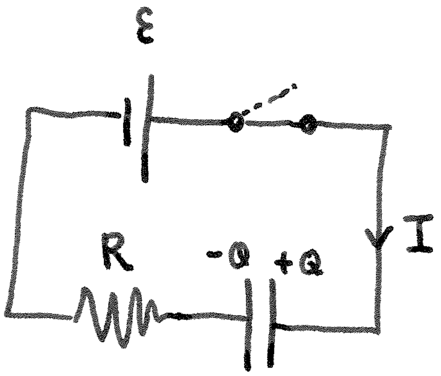
e.g. $C = 2.6\,\mu F$ $\quad R = 2\,M\Omega \Rightarrow \tau = \underline{\underline{5.2\,s}}.$



Charging



$$\varepsilon - Q/C - IR = 0 \qquad \boxed{\frac{dQ}{dt} = \frac{\varepsilon}{R} - \frac{Q}{CR}}$$

$I = dQ/dt$

$$I_0 = \frac{\varepsilon}{R} \qquad Q_{final} = \varepsilon C.$$

$$\frac{dQ}{dt} = \frac{1}{CR}\left(\varepsilon C - Q\right)$$

Solution :

$$\frac{dQ}{C\varepsilon - Q} = \frac{dt}{CR}$$

$$\int_0^Q \frac{dQ'}{C\varepsilon - Q'} = \frac{t}{CR}$$

$$\|$$

$$\ln\left(\frac{C\varepsilon}{C\varepsilon - Q}\right) = \frac{t}{\tau} \qquad \tau = CR$$

$$\frac{C\varepsilon}{C\varepsilon - Q} = e^{t/\tau}$$

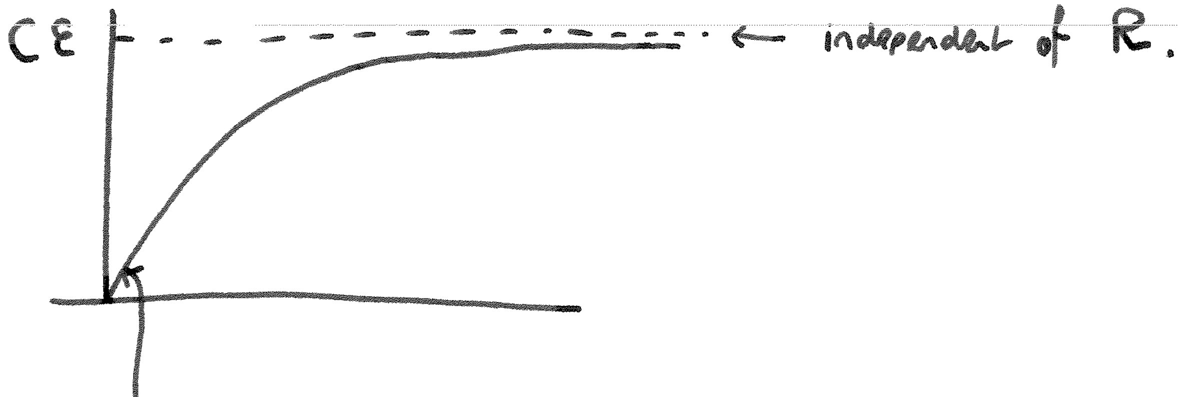
$$1 - \frac{Q}{C\varepsilon} = e^{-t/\tau}$$

$$\frac{Q}{C\varepsilon} = 1 - e^{-t/\tau}$$

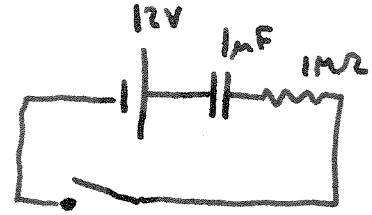
$$Q = \overbrace{C\varepsilon}^{Q_0}\left(1 - e^{-t/CR}\right)$$

$$I = \frac{dQ}{dt} = \underbrace{\frac{\varepsilon}{R}}_{I_0} e^{-t/CR}$$

$$\boxed{\begin{aligned} Q &= Q_0\left(1 - e^{-t/\tau}\right) \\ I &= I_0 e^{-t/\tau} \end{aligned}}$$



$I_0 = \frac{\mathcal{E}}{R}$ (independent of C)



e.g. Resistor with $R = 1\,M\Omega$ in series + $1\,\mu F$ with $\mathcal{E} = 12V$. Before switch is closed $Q = 0$.

a) What is the time constant? What is initial current I_0?

b) What fraction of the initial current remains after 5s.

c) How long before $I/I_0 = 1/2$?

a) $\tau = CR = 10^{-6} \times 10^{6}\,\Omega = \underline{1\,s}$. $\quad I_0 = \mathcal{E}/R = \underline{12 \times 10^{-6}\,A}$

b) $I/I_0 = e^{-t/\tau} = e^{-5} = \underline{6.74 \times 10^{-3}}$

c) $I/I_0 = 1/2 = e^{-t/\tau} \Rightarrow \ln(0.5) = -t/\tau$

$t = -\tau \ln 0.5 = \underline{6.93\,s}$