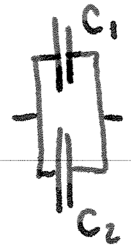
L11 D.C CIRCUITS

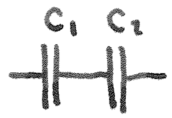
Apart from their use in lightbulbs, resistors are one of the basic elements in circuitry, and their function is to control the current & voltage passed through a device.

To understand, use and ultimately to design electrical circuits, we need to learn how they work in complex combinations. Before we can understand circuits in which the current is time dependent, we need to understand circuits with constant or "direct" current, with its well-known acronym "D.C."

Recall Capacitors in Series & Parallel

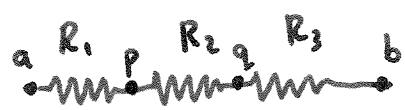


$C = C_1 + C_2$ (same voltage)

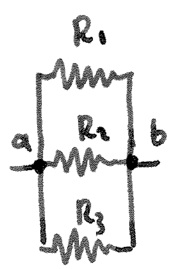


$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ (same charge)

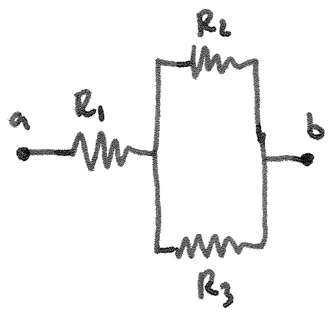
now we need to consider similar combinations of resistors



series

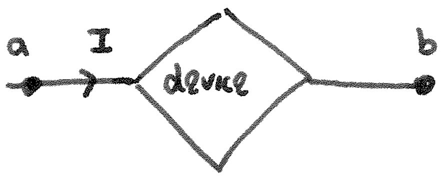


parallel



R_1 in series with parallel combination of R_2 & R_3.

Concept of equivalent resistance



$$R_{eq} = \frac{V_{ab}}{I}$$

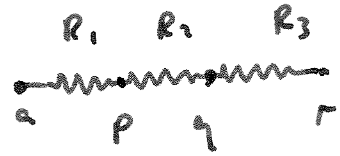
Series

$$V_{ap} = IR_1 \qquad V_{pq} = IR_2 \qquad V_{qb} = IR_3$$

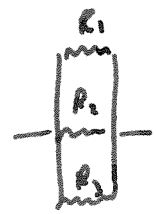
$$V_{ab} = V_{ap} + V_{pq} + V_{qb}$$

$$= I\left(R_1 + R_2 + R_3\right)$$

$$\boxed{R_{eq} = \frac{V_{ab}}{I} = R_1 + R_2 + R_3}$$



Parallel



$$V_{ab} = I_1 R_1 = I_2 R_2 = I_3 R_3$$

$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\boxed{\frac{1}{R_{eq}} = \frac{I}{V_{ab}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

Two resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

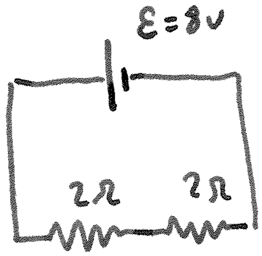
$$V_{ab} = I_1 R_1 = I_2 R_2 \Rightarrow \frac{I_2}{I_1} = \frac{R_1}{R_2}$$ inversely proportional to resistance

e.g. Two 2Ω light bulbs, connected in series & in parallel to a $\mathcal{E} = 8V$ source

a) Find current, voltage & power delivered to each bulb in series

b) parallel

c) Suppose one bulb burns out, what happens to the other?

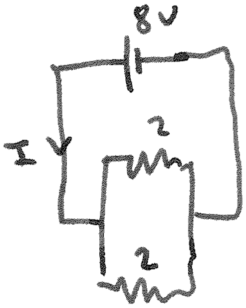
a) 

$R_{eq} = 2+2 = 4\Omega$

$I = \frac{8}{4} = 2A$

$V = 4V$

$P_1 = P_2 = I^2R = (2)^2 \times 2 = 8W$

b) 

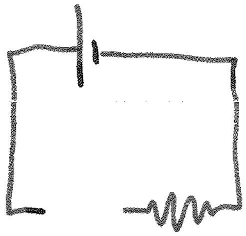
$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{2} = 1 \Rightarrow R_{eq} = 1\Omega.$

$I_{TOT} = \frac{\mathcal{E}}{R_{eq}} = \frac{8V}{1\Omega} = 8A$

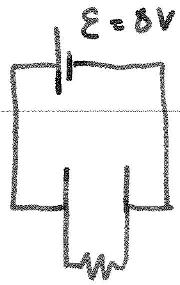
$I_1 = I_2 = \frac{8}{2} = 4A. \quad V_1 = V_2 = 8V$

$P_1 = P_2 = 4 \times 8V = \underline{32 W}$ ✓ $P_{TOT} = \mathcal{E}I$
$= 8 \times 8A = 64W$

c)



Series: one bulb burns out, then second bulb goes out



R_{eq} goes from $R_{eq} = 1\Omega$ to $R_{eq} = 2\Omega$

I_1 goes from $4A$ to $4A$.

Unchanged brightness.

26.2 Kirchoff's Rules

When we have more complex circuits, we can no longer rely on the elimination of series or parallel combinations by their effective resistance.

A more general set of rules is required —

Kirchoff's Rules

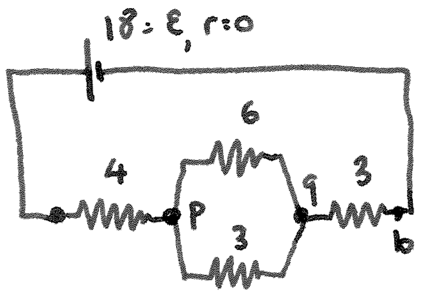
1. $\sum I = 0$ at each junction (JUNCTION RULE)

2. $\sum_{\text{loop}} V = 0$ valid for each closed loop (LOOP RULE)

"1" says that current & charge are conserved.

"2" is the statement that the electrostatic force is conservative — when you go around a loop you arrive back at the same voltage.

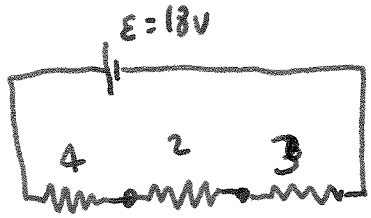
Systematic elimination of series + parallel combinations.



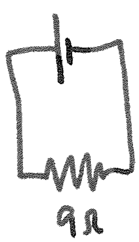
a) Find equivalent resistance R_{eq}

b) Find current in each resistor.

$\equiv$ $\frac{1}{3} + \frac{1}{6} = \frac{1}{2} \Rightarrow R'_{eq} = 2\Omega$



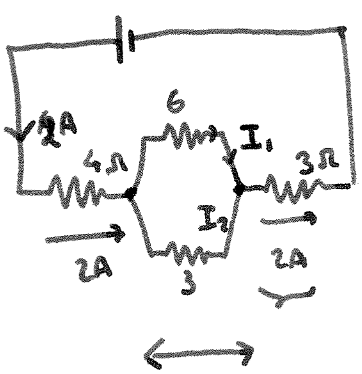
$\equiv$



$4 + 2 + 3 = 9\Omega$

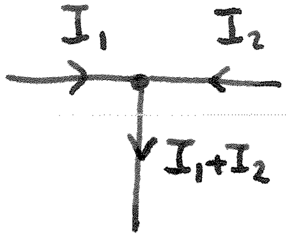
$R_{eq} = 9\Omega$

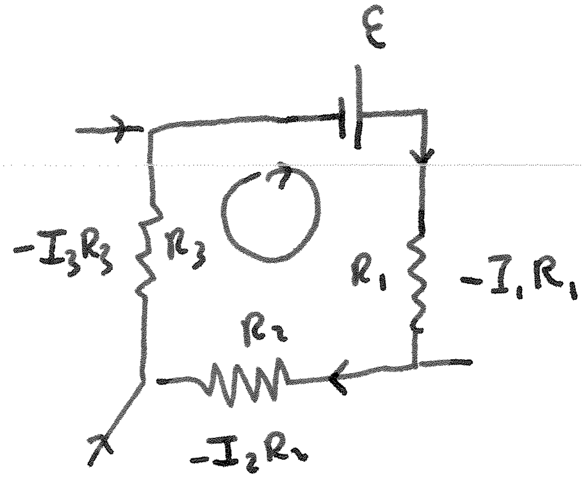
$I = \frac{\varepsilon}{R_{eq}} = \frac{18}{9} = \underline{2A}$.



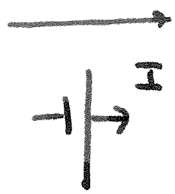
$$V_{pq} = 18 - 7 \times 2 = 4V$$

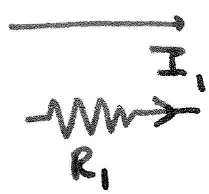
$$\left.\begin{array}{l} I_2 = \frac{4V}{3\Omega} = \frac{4}{3}\,\Omega \\ I_1 = \frac{4V}{6\Omega} = \frac{2}{3}\,\Omega \end{array}\right\} \; I_1 + I_2 = \frac{4+2}{3} = 2A \checkmark.$$





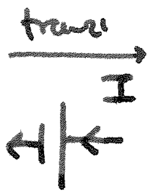
$$\mathcal{E} - I_1R_1 - I_2R_2 - I_3R_3 = 0$$

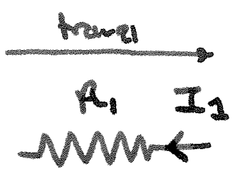




$+\mathcal{E}$

$-I_1R_1$.

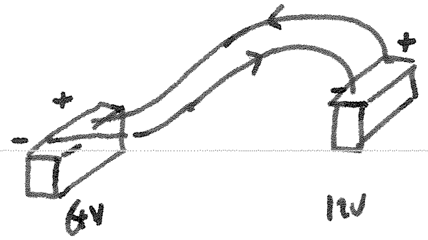




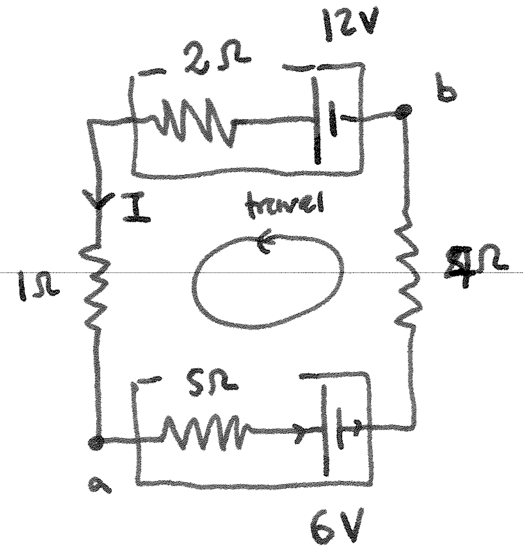
$-\mathcal{E}$

$+I_1R_1$

Ex 26.3 Single loop



"Shunting" a battery.



Find

a) current I in circuit

b) V_{ab}

c) power output of the EMF of each battery.

c.f. "shunting a battery"

a) $12V - I\times 2 - I\times 1\Omega - I\times 5\Omega - 6V - I\times 4\Omega = 0$

$$6V - I\times(\overbrace{2+1+5+4}^{12\Omega}) = 0 \quad \Rightarrow \quad I = 0.5A.$$

b) $V_{ab} = +5\times 0.5 + 6 + 4\times 0.5 = 9\times 0.5 + 6$

$= 4.5 + 6 = 10.5V$

check

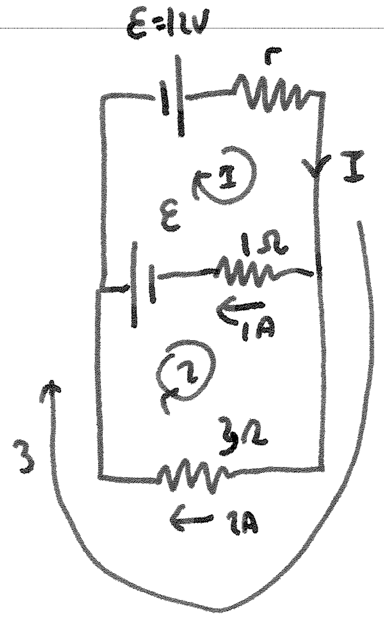
$V_{ab}: \quad V_a - V_b = +12 - 2(0.5) - 1\times(0.5) = 12 - 1.5 = 10.5V.$

c) $P_{12V} = \mathcal{E} I = 12\times 0.5 = 6W$

$P_{6V} = -\mathcal{E} I = -6\times 0.5 = 3W.$

ex. **CHARGING A BATTERY** 26.4

a) Find the unknown current, internal resistance & EMF $\mathcal{E}$



Kirchoff I $\Rightarrow$ $I = 1A + 2A = 3A$

Loop 1) $12V - Ir - 1\times1 + \mathcal{E} = 0$

$11 - 3r + \mathcal{E} = 0.$

Loop 3) $12V - Ir - 2A \times 3\Omega = 0$

$6 - 3r = 0$

$\Rightarrow r = 2\Omega.$

from loop 1) $11 - 3.2 + \mathcal{E} = 5 + \mathcal{E} = 0$

$\Rightarrow \mathcal{E} = -5\,V.$

b) Find power output by each battery.

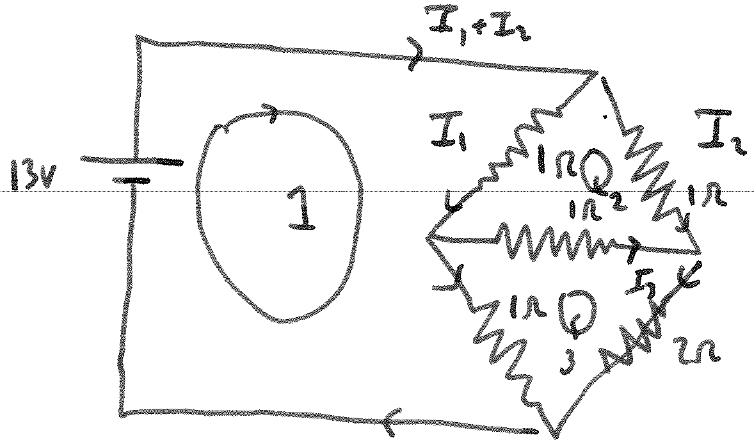
$P_{12V} = 12 \times 3 = 36W$

$P_{bulb} = (2)^2 \times 3\Omega = 12W$

$P_{\mathcal{E}} = -5 \times 1 = -5W$

$P_{r\text{-}battery} = I^2 r = (1A)^2 \times 1\Omega = \underline{1W}.$

Complex network



$I = I_1 + I_2$

① $13V - I_1 1 - (I_1 - I_3).1 = 0 \Rightarrow 13 - 2I_1 + I_3 = 0$

② $-I_2 . 1 + I_3 . 1 + I_1 . 1 = 0 \Rightarrow I_2 = I_1 + I_3.$

③ $-I_3 . 1 - (I_2 + I_3)2 + (I_1 - I_3) . 1 = 0$

$\Rightarrow -2I_2 - 4I_3 + I_1 = 0$

$\Rightarrow -2(I_1 + I_3) - 4I_3 + I_1 = -6I_3 - I_1 = 0$

$\underline{I_1 = -6I_3}.$

$13 - 2(-6I_3) + I_3 = 13 + 12I_3 + I_3 = 13 + 13I_3 \Rightarrow \underline{I_3 = -1A}$

$I_1 = 6A$

$I_2 = I_1 + I_3 = 6 - 1 = \underline{\underline{5A}}$

$$R_{eq} = \frac{13V}{I_1 + I_2} = \frac{13}{11} \simeq \underline{1.2\Omega}$$