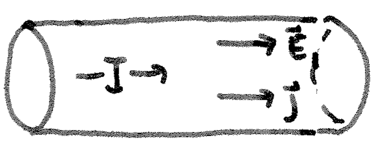
L10 Circuits, E.M.F., Power & Model of resistance

Last time we learned that the fluid of electricity is made up of electrons or holes, moving at an incredibly high velocity. The scattering of these electrons inside the metal means that their average drift velocity is only a fraction of a mm per second.

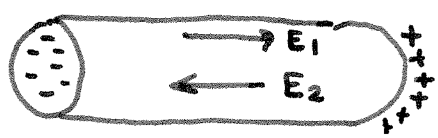
Today we will discuss how we set up a constant current inside a metal, and this will involve the concept of a <u>circuit</u> – literally a closed "loop" around which electrical current can flow. We'll also learn about "Electro-motive force" – actually the driving voltage that pumps charge around a circuit.

25.4

In order for electrons to flow, they need a complete circuit. If a circuit is broken, charge quickly builds up a the end of the wire, producing an electric field which cancels the external electri field



a)

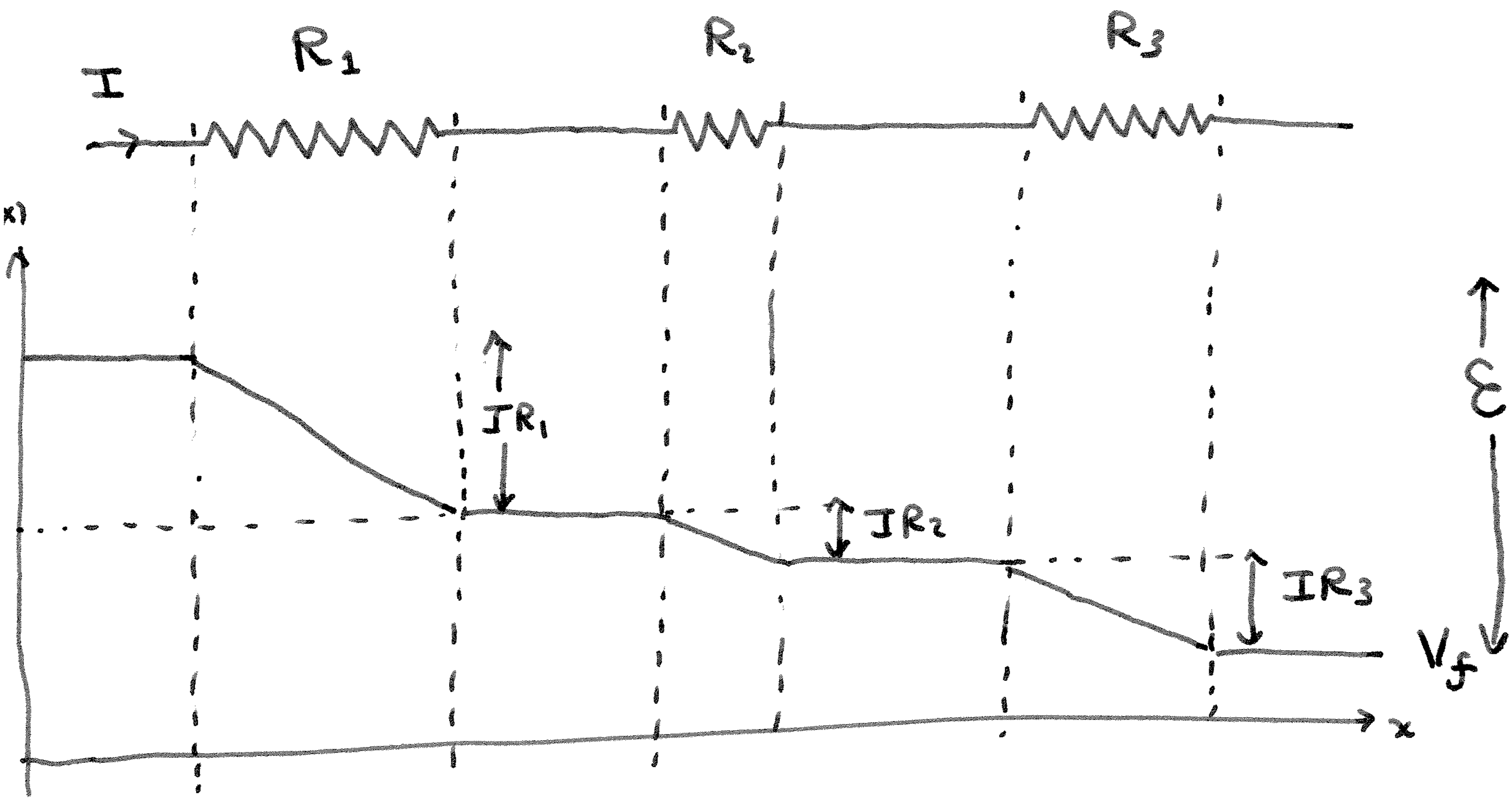


b) Build up of charge at ends of broken circuit produces a field $\vec{E}_2$ which exactly cancels the external field.

25.4 E.M.F.

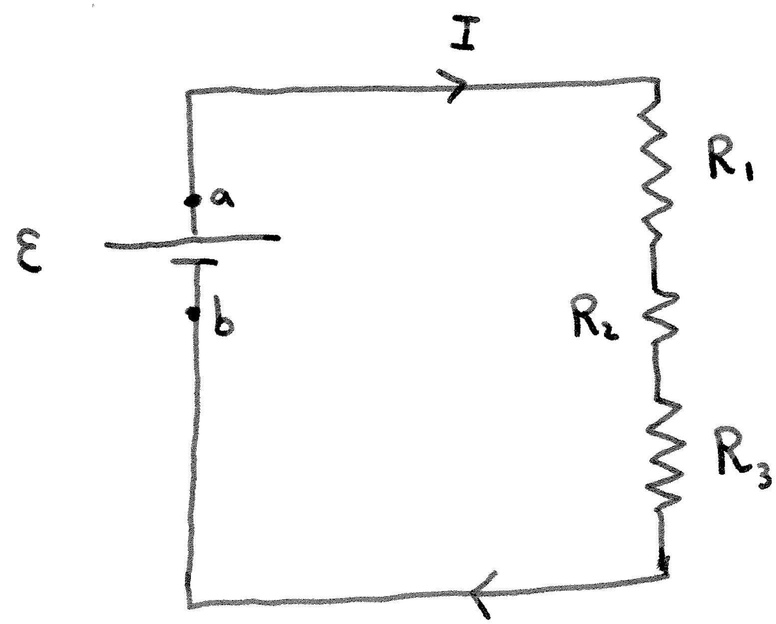
When a current flows through a resistor, it flows from "high" to "low" voltage. The voltage drop is given by

$$V_{ab} = IR$$



Its a little like running "downhill".

At some point one needs a "pump" in the circuit to bring the charged particles back up to the starting potential. The difference between the starting & finishing potential is the electro-motive force, or E.M.F. In a circuit diagram we would draw this as follows



$$\boxed{V_{ab} = \varepsilon}$$

and we write

$R = R_1 + R_2 + R_3$

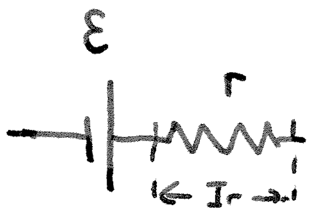
$$\varepsilon = IR_1 + IR_2 + IR_3 \;. = I\,R$$

Internal resistance

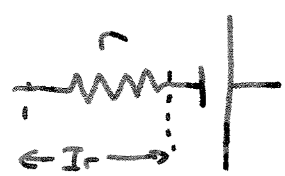
In practice, all E.M.F sources (batteries, solar cells, thermocouples, fuel cells) have an internal resistance. We label this as "r". When a current flows, the internal resistance r reduces the output voltage

$$\boxed{V_{ab} = \mathcal{E} - Ir}$$

We denote such a battery by the circuit diagram



or

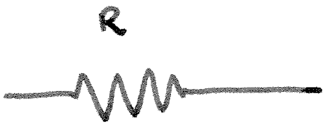


$$\mathcal{E} = \underbrace{Ir}_{\text{internal resistance}} + \underbrace{IR}_{\text{external resistance}}$$

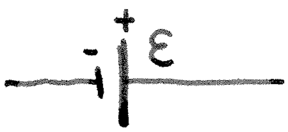
$$\boxed{I = \frac{\mathcal{E}}{R + r}}$$

Symbols for Circuit Diagrams

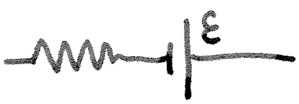
conductor with negligible resistance



resistor

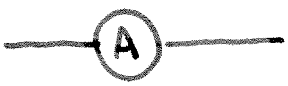


E.M.F. source

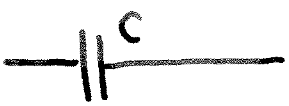


E.M.F source + internal resistance

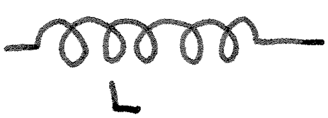
Voltmeter (measures potential difference)



Ammeter (measures current through it)



Capacitor C



Inductor L

Josephson Junction.

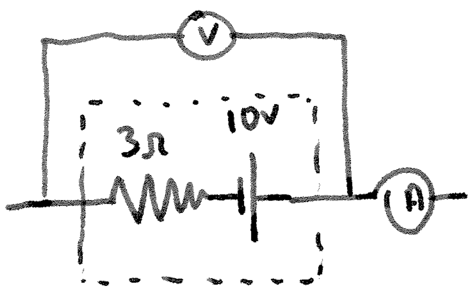
e.g. Source on open circuit. Battery with EMF $\varepsilon = 10V$ & internal resistance $r = 3\Omega$. What is

a) current + voltage in an open circuit?

b) current + voltage when connected in series with a 7Ω resistor?

c) current & voltage when short-circuited?

a)

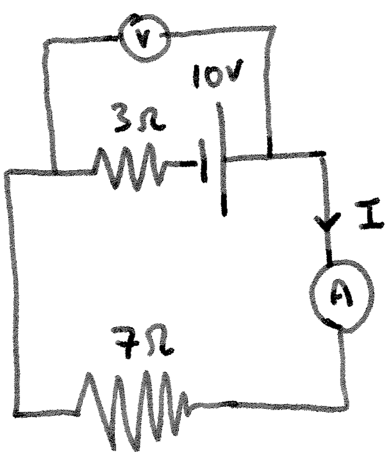


<u>Open circuit</u>

$I = 0$ on Ammeter

$V_{ab} = \varepsilon - I\cancel{R} = \varepsilon = 10V$

b)



$$\varepsilon = I(\overset{3}{r} + \overset{10}{R})$$

$$= I(10\Omega)$$

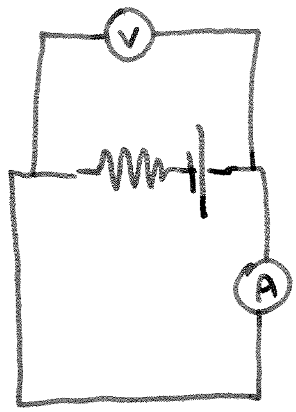
$$\Rightarrow I = \frac{\varepsilon}{10} = \underline{1A}.$$

$$V_{ab} = \varepsilon - Ir$$

$$= 10 - 1 \times 3\Omega$$

$$= \underline{7V}$$

c)



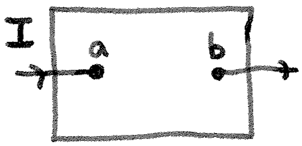
<u>Short circuit</u>

$$I = \frac{\varepsilon}{r+R} = \frac{\varepsilon}{r} = \frac{10\,V}{3\,\Omega} = 3.\dot{3}\,A$$

$$V = \varepsilon - Ir = 0.$$

25.5 Energy + Power

Work done on device $= dq\,V_{ab} = dq(V_a - V_b) = dW$



$$dq = I\,dt$$

$$dW = dq\,V_{ab} = I\,V_{ab}\,dt$$

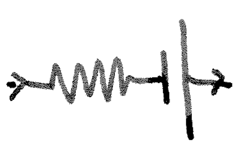
$$\boxed{P = \frac{dW}{dt} = I\,V_{ab}}$$

$(1\,C/s)(1\,J/C) = 1\,J/s = 1\,W$

WATT

For a pure resistor, $\boxed{P = I V_{ab} = I^2 R = \frac{V_{ab}^2}{R}}$

Source – power output.



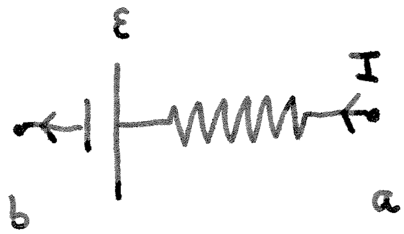
$$P = I\,V_{ab}$$
$$= I(\mathcal{E} - Ir)$$
$$= \mathcal{E}I - I^2 r$$

loss to internal resistance

Source - power input (e.g charging laptop battery).

$$V_{ab} = \mathcal{E} + Ir$$

$$P = V_{ab} I = \mathcal{E} I + I^2 r$$



$$\mathcal{E} I = P - I^2 r$$

input power

loss to internal resistance.

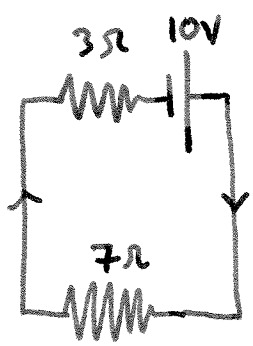
rate of conversion from electrical to chemical energy.

e.g. Power output from a source. Find

(a) rate of energy conversion (chemical to electrical)

(b) rate of dissipation of energy in battery.

(c) power output.



$$I = \frac{\mathcal{E}}{r+R} = \frac{10}{10} = 1A$$

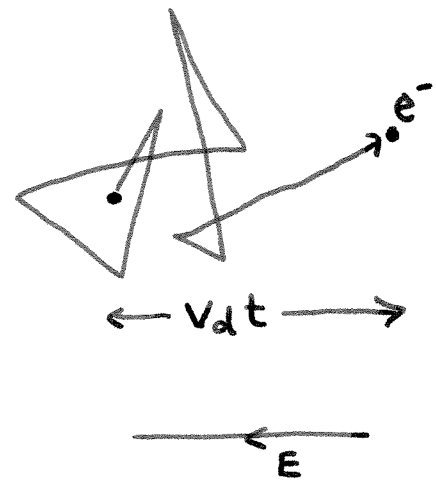
a) $\mathcal{E} I = 10 \times 1 = 10W$

b) $I^2 r = (1 \times 1) \times 3 = 3W$

c) $P = I^2 R = 7W.$

25.6 Drude Model for Resistivity

In an electric field, electrons acquire a drift velocity $\vec{v}_d$. The current density is then



$$\vec{J} = nq\vec{v}_d$$

Now we know that the electron accelerates in a field with an acceleration

$$\vec{a} = \frac{\vec{F}}{m_e} = \frac{q\vec{E}}{m_e}.$$

The velocity after a time t is

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

The average of this quantity, v_d = <v> = <vo> + **a** <t>, is the drift velocity.

But the average velocity after a collision is zero <vo>=0 and the average time between collisions <t>=τ, the collision time, so

$$\vec{v}_d = \vec{a}\tau = \frac{q\vec{E}}{m}\tau$$

and hence the current density is

$$\vec{J} = \frac{nq^2\tau}{m}\vec{E}$$

But $\vec{J} = \frac{1}{\rho}\vec{E}$ where ρ is the resistivity, so

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$ DRUDE FORMULA.

e.g. Calculate the scattering time τ for silver, where $\rho = 1.47 \times 10^{-8}\ \Omega\,\mathrm{m}$ and $n_e = 5.9 \times 10^{28}\ \mathrm{m}^{-3}$.

$$\rho = \frac{m}{ne^2\tau} \Rightarrow \tau = \frac{m}{ne^2\rho}$$

$$\tau = \frac{9.1 \times 10^{-31}\,\mathrm{kg}}{(5.9 \times 10^{28}\,\mathrm{m}^{-3})(1.6 \times 10^{-19})^2(1.47 \times 10^{-8}\,\Omega\,\mathrm{m})}$$

$$= 4.1 \times 10^{-14}\,\mathrm{s}$$

About 2.5×10^{13} or

25,000,000,000,000

collisions per second!